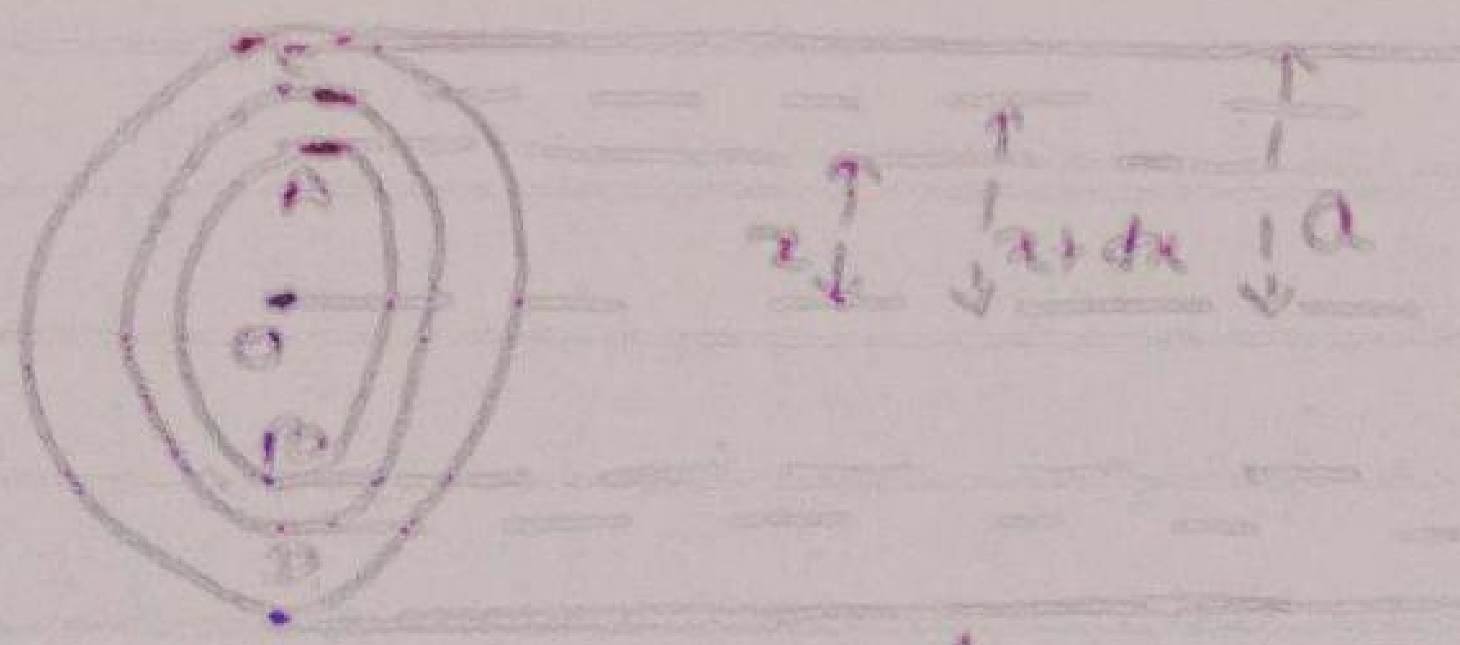
2016 Que Derive expression for flow of a compressible fluid through a narrow tube. Discuss the effect of pressure and temperature on viscosity coefficient of gases.

When the tube be narrow and the velocity of liquid flow is quite small, the following assumptions are ~~found~~ valid.

(i) The liquid flow is steady or streamline, with the streamlines parallel to the axis of the tube.

(ii) Since there is no radial flow, the pressure, in accordance with Bernoulli's theorem, is constant over any given cross section of the tube.

(iii) The liquid in contact with the walls of the tube is stationary.



P = A liquid flows through a narrow tube under a pressure diff P.

l = length of capillary tube.

a = Radius of capillary tube

u = velocity of flow of liquid at all points on an imaginary coaxial cylindrical shell of the liquid, of radius x

du/dx = velocity gradient (velocity of flow the liquid is max along the axis of the tube and decreases to zero at its walls.)

Let us consider the co-axial liquid cylinder of radius x. The velocity of flow at all points on the surface liquid cylinder is the same. In accordance with Newton's law of viscous flow, the backward dragging force on the imaginary liquid shell is

$$F = -\eta A\, du/dx = \eta \times 2\pi x l \left[-du/dx\right]$$

The force on the liquid shell, accelerating it forwards $= P\pi x^2$

For the liquid-flow to be steady, the driving force equal to backward dragging force

$$P\cancel{\pi} x^{\cancel{2}} = -2\cancel{\pi}\cancel{x} l\,\eta\, du/dx$$ (-ve sign the dragging force acts opposite to driving force)

$$Px = -2l\eta\, du/dx$$

$$du = -\frac{Px\,dx}{2l\eta}$$

Integrating it

$$u = -\frac{P}{2\eta l}\int x\,dx = -\frac{P}{2\eta l}\left[\frac{x^2}{2}\right] + C_1$$

C_1 is integration constant

$x = a \quad u = 0$

$$0 = -\frac{Pa^2}{4\eta l} + C_1 \qquad C_1 = \frac{Pa^2}{4\eta l}$$

$\therefore$ velocity of flow at distance x from the axis of the tube i.e.

$$u = -\frac{Px^2}{4\eta l} + \frac{Pa^2}{4\eta l} = \frac{P}{4\eta l}(a^2 - x^2)$$

$$a^2 - x^2 = \frac{4\eta l}{P}u$$

The volume dv of liquid which flows through the tube in unit time between the radii x and $x+dx$

$$dv = 2\pi x\,dx\cdot u = \frac{P\pi}{2\eta l}(a^2 - x^2)\,r\cdot dr$$

The total volume V flowing through the tube in unit time.

$$\int dV = \int_0^a \frac{P\pi}{2\eta l}(a^2 - x^2)\,x\,dx = \frac{P\pi a^4}{8\eta l}$$

Flow of a compressible fluid through a narrow Tube →

For a compressible fluid ie gases the mass which crosses any section in a given time is constant

$$\rho \alpha \omega = \text{constant}$$

where ρ is the density at a point in the section, area α of a tube of flow and ω is velocity.

$\alpha\omega$ = The volume is flowing through the area per second.

~~dx~~ = consider an element of the tube of length dx

dp = The difference in pressure between its end.

V = The volume of the element is flowing per second.

$$V = -\frac{\pi a^4}{8\eta} \cdot dP/dx$$

P_1 = pressure at the inlet end.

V_1 = The volume entering the tube per second.

then, the cross-sectional area is constant.

$$P_1 V_1 = PV$$

$$P_1 V_1 = -\frac{P \pi a^4}{8\eta} dP/dx$$

so that

$$\int_0^l P_1 V_1 dx = -\frac{\pi a^4}{8\eta} \int_{P_1}^{P_2} P dP$$

where P_2 is the pressure at the outlet end of the tube, Hence.

$$\boxed{P_1 V_1 = \frac{(P_1^2 - P_2^2)\pi a^4}{16\eta l}}$$

Grindey and Gibson noted that the diff of pressure between the two ends forcing water into the container, and V the volume of ~~water~~ gas passing through the tube per second then η for the gas was evaluated.

Correction to Poiseuille's equation →

In the Poiseuille's equation two important corrections are to be applied

(I) It has been assumed that the liquid does not possess any K.E at the outlet end of the tube. But the liquid flowing out of the tube has a ~~certain~~ certain velocity and hence possess K.E so effective pressure

$$P_1 = P - P'$$

$$K.E = E' = \int_0^r \frac{1}{2} m u^2 = \frac{1}{2}\int_0^r \left[(2\pi x\, dx)\, u \rho\right] u^2$$

$$E' = \pi \rho \int_0^r u^3 x\, dx = \pi \rho \int_0^r \left(\frac{P}{4\eta l}\right)^3 (r^2 - x^2)^3 x\, dx$$

$$u = \frac{P(r^2 - x^2)}{4\eta l}$$

$$E' = \pi \rho \left(\frac{P}{4\eta l}\right)^3 \int_0^r (r^6 x - 3r^4 x^3 + 3r^2 x^5 - x^7)\, dx$$

$$E' = \pi \rho \left(\frac{P}{4\eta l}\right)^3 \left(\frac{r^8}{8}\right) = \left(\frac{\pi P r^4}{8\eta l}\right)^3 \frac{\rho}{\pi^2 r^4}$$

$$\boxed{E' = \frac{V^3 \rho}{\pi^2 r^4}}$$

$$P' V = E'$$

$$P' = \frac{E'}{V} = \frac{V^2 \rho}{\pi^2 r^4}$$

$$\boxed{P_1 = P - \frac{V^2 \rho}{\pi^2 r^4}}$$

If V is small Then $\boxed{P_1 = P}$

(II) At the inlet end ~~of the~~ tube, the flow of liquid is not streamline for some distance consequently the liquid is accelerated. The effective length of tube $= (l + 1.64r)$

$$\eta = \frac{\pi P r^4}{8V(l+1.64r)} - \left(\frac{V^2 \rho}{\pi^2 r^4}\right) \frac{\pi r^4}{8V(l+1.64r)}$$

$$\boxed{\eta = \frac{\pi P r^4}{8V(l+1.64r)} - \frac{V\rho}{8\pi(l+1.64r)}}$$

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